

Quadratic Forms and Motives

The objective of this joint seminar is to serve as a first part of a long term project on understanding material and ideas leading to a proof of Milnor’s conjecture. The prototype of our program plan is the program of the summer school “Motives and Milnor conjecture”, held in Paris in 2011.

Ideally, each talk should last **90–120 minutes** with a 10-minute break if needed.

Part I: Basics on Chow Motives

Talk 1: *Chow groups in terms of K -homology and K -cohomology following Rost*

In this talk, we develop the K -homology and K -cohomology theories of schemes that generalizes the theory of Chow groups following the approach of Rost. It has several advantages comparing to the theory developed by Fulton. It has a long localization exact sequence and it simplifies the construction of Gysin pullback avoiding the intersection with Cartier divisors.

- Goal : Define Rost complex $C_*(X)$ in terms of Milnor K -groups.

Sketch the basic properties for push-forward and pullback homomorphisms.

Define the K -homology group and explain why it agrees with the usual Chow group modulo rational equivalence at the given degree.

Deduce the usual properties on push-forward and pullback.

Sketch the deformation homomorphisms for K -homology groups.

Deduce the projective bundle formula and the properties of Chern classes.

Sketch the K -cohomology ring of smooth schemes.

If time permits, mention the definitions and properties of Segre classes.

Try to skip all technical proofs and focus on main definitions, constructions and statements of results.

- Prerequisites : general knowledge of algebraic geometry

- References :

[EKM08] The algebraic and geometric theory of quadratic forms, [Chapter IX](#), [Chapter X](#), [Appendices](#).

[Ful98] Intersection theory.

[Ros96] Chow groups with coefficients.

Talk 2: *Chow motives*

- Goal : Cover the whole Chapter XII of [EKM08]. Explain the constructions of the categories of correspondences and Chow motives. Prove the nilpotence theorem.
- Prerequisites : general knowledge of algebraic geometry; basic notions and facts from Talk 1
- References :
[EKM08] The algebraic and geometric theory of quadratic forms, Chapter XII
[Man68] Correspondences, motifs and monoidal transformations.

Talk 3: *Cycles on powers of quadrics*

- Goal : Cover the whole Chapter XIII of [EKM08]. Describe the Chow group of quadrics and the reduced Chow group on X^r .
In the longest section §73, you can make your choice of results to talk about and results to skip.
- Prerequisites : general knowledge of algebraic geometry; basic notions and facts from Talks 1 and 2
- References :
[EKM08] The algebraic and geometric theory of quadratic forms, Chapter XIII
[Kah06] Formes quadratiques et cycles algébriques (d'après Rost, Voevodsky, Vishik, Karpenko et al.)
[Vis08] Topics on quadratic forms.
[Vis04] Motives of quadrics with applications to the theory of quadratic forms.

Part II: Supplementary Talks

Talk 4: *Quadratic forms in characteristic 2*, by 吴正尧

The goal of this talk is to offer the audience an introduction to the theory of quadratic forms in characteristic 2.

Talk 5: *Symbol lengths and generic splittings in Galois cohomology*, by 孙鹏

The goal of this talk is to explain the main results of Krashen's work [Kra16].

References

- [EKM08] Richard Elman, Nikita Karpenko, and Alexander Merkurjev. *The algebraic and geometric theory of quadratic forms*, volume 56 of *American Mathematical Society Colloquium Publications*. American Mathematical Society, Providence, RI, 2008.
- [Ful98] William Fulton. *Intersection theory*, volume 2 of *Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics]*. Springer-Verlag, Berlin, second edition, 1998.
- [Kah06] Bruno Kahn. Formes quadratiques et cycles algébriques (d’après Rost, Voevodsky, Vishik, Karpenko et al.). *Astérisque*, (307):Exp. No. 941, vii, 113–163, 2006. Séminaire Bourbaki. Vol. 2004/2005.
- [Kra16] Daniel Krashen. Period and index, symbol lengths, and generic splittings in Galois cohomology. *Bull. Lond. Math. Soc.*, 48(6):985–1000, 2016.
- [Man68] Ju. I. Manin. Correspondences, motifs and monoidal transformations. *Mat. Sb. (N.S.)*, 77 (119):475–507, 1968.
- [Ros96] Markus Rost. Chow groups with coefficients. *Doc. Math.*, 1:No. 16, 319–393, 1996.
- [Vis04] Alexander Vishik. Motives of quadrics with applications to the theory of quadratic forms. In *Geometric methods in the algebraic theory of quadratic forms*, volume 1835 of *Lecture Notes in Math.*, pages 25–101. Springer, Berlin, 2004.
- [Vis08] Alexander Vishik. Topics on quadratic forms. In *Some recent developments in algebraic K-theory*, volume 23 of *ICTP Lect. Notes*, pages 228–275. Abdus Salam Int. Cent. Theoret. Phys., Trieste, 2008.